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RELATION OF SLUG STABILITY TO SHEDDING RATE

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Abstract—Measurements are presented for the shedding rate of liquid from slugs created by the flow of air and water in a horizontal 0.0953 m pipe at atmospheric conditions. These are used to predict a critical liquid carpet height below which slugs will decay. Of particular interest is the finding that the initiation of the slug flow regime at high gas flows is related to the stability of slugs, rather than the stability of a stratified flow. Copyright © 1996 Elsevier Science Ltd.

Key Words: gas–liquid pipe flow, slug flow, stability of slugs, stability of stratified flows

1. INTRODUCTION

A number of regimes are exhibited when gas and liquid flow through a horizontal pipe. Two intermittent patterns are commonly defined, plug and slug flow. The plug pattern is found at very low gas throughputs; it is characterized by elongated bubbles that move along the top of the pipe between liquid plugs that do not contain air bubbles. If the gas throughput at a given liquid flow is increased, a transition from plug to slug flow occurs. This transition is associated with the appearance of a hydraulic jump, at the front of the liquid plug, which entrains air (Ruder & Hanratty 1990). Thus, slug flow is characterized by the intermittent appearance of highly aerated slugs of liquid that travel down the pipeline approximately at the local gas velocity, and are separated from one another by a stratified configuration of the gas and liquid phases.

The prediction of the flow conditions at which slugs will be observed has received considerable attention in the last two decades. One approach is to analyze the stability of a stratified flow. Kordyban & Ranov (1970) suggested that the transition from a stratified flow to a slug flow may be described through a classical linear stability analysis. Wallis & Dobson (1973), Lin & Hanratty (1986) and Wu *et al.* (1987) examined the growth of linearly unstable long wavelength disturbances on a flowing liquid. The evolution of a slug from a finite amplitude wave, with a wavelength in a range that would be considered stable by the Kelvin Helmholtz mechanism, has been considered by Kordyban & Ranov (1970), Taitel & Dukler (1976), Mishima & Ishii (1980) and by Fan *et al.* (1993).

Another approach is to examine the stability of slugs traveling over a liquid layer. Dukler & Hubbard (1975) pictured a stable slug as one for which liquid is scooped from the slower moving liquid carpet at the slug front at the same rate as liquid is shed from the tail. Conservation of mass arguments can be used to show that the rate at which liquid is picked up at the front is governed by the height, h_{L1}, of the liquid layer. Ruder *et al.* (1989) developed conditions for the existence of slugs by arguing that the liquid volumetric shedding rate at the tail of a slug, Q_L, may be calculated by considering the back of a slug to be the same as the inviscid bubble resulting from the penetration of air into liquid draining from a horizontal tube (Benjamin 1968). The shedding velocity Q_L/A is predicted to be proportional to the square root of the product of the gravitational acceleration and the pipe diameter, $\sqrt{gD}$. The outcome of the analysis by Ruder *et al.*, is the prediction of a critical height of the liquid carpet that is needed to ensure the stability of a slug. This prediction of a minimum h_{L1} is independent of the gas density and requires the assumption of negligible effects of aeration and surface tension.

For air and water at atmospheric pressure and at low gas velocities, a stratified flow becomes unstable at liquid layer heights that are larger than the critical value required for a stable slug to exist. A consequence of this is that slugs can occur at lower liquid flows than would be predicted by a stability analysis of the stratified flow if a disturbed entry were used. Experiments by Salkudean *et al.* (1983) and by Bendiksen & Malnes (1987) support this conclusion. Furthermore, Ruder *et al.* (1989), Hanratty (1991) and Bendiksen & Espedal (1992) point out that for very high gas densities, stratified flows become unstable at lower liquid heights than is required for a stable slug to exist. This suggests that correlations for the initiation of slugs at large gas densities should evolve from a consideration of the necessary conditions for the existence of slugs, rather than a stability analysis of the stratified flow.

Substantiation of the necessary conditions developed by Ruder *et al.* (1989) and by Bendiksen & Espedal (1992) would be advanced if direct measurements of the shedding rate, Q_L, were made under slug flow conditions. Some progress along this line has been made by Bendiksen (1984), who injected gas bubbles into a liquid stream, and by Fan *et al.* (1992), who studied stationary slugs. Fan *et al.* concluded that the tail of a stationary slug is an inviscid bubble only if the liquid volumetric flow rate out the tail of the slug is very low.

This paper presents measurements of Q_L for slugs that result from air and water flowing in a horizontal 0.0953 m pipeline at atmospheric conditions. Conductance profiles were used to determine the liquid holdup at several locations along the pipeline. These conductance measurements establish the profiles of the liquid carpet and the tail of the slug, and the degree of aeration in a slug. By using a translating control volume attached to the back of the slug, conservation of liquid volume is used to calculate Q_L. An accumulation term is included in the conservation equation in order to take into account changes in the liquid volume that occur in the control volume as it translates downstream. In this manner Q_L is calculated for stable, growing and decaying slugs.

A goal of this paper is to predict the initiation of slugs at high gas velocities. Studies by Lin & Hanratty (1986), Andritsos *et al.* (1989) and by Fan *et al.* (1993) show that the initiation of slugging in air–water systems at atmospheric conditions cannot be predicted from a consideration of the stability of a stratified flow at superficial gas velocities greater than 4 m/s. In fact, slugs at high gas velocities appear to form through the coalescence of waves when the height of the stratified flow is large enough. Measurements of Q_L are used to explain this transition in terms of the stability of a slug rather than the stability of a stratified flow.

2. PHYSICAL DESCRIPTION OF THE SLUG

Parameters characterizing a slug are given in figure 1. The tail, station 4, translates at the bubble velocity C_B. Station 2 denotes the front, which moves at velocity C_F. Air is incorporated into the slug at station 2 at low gas velocities. The bubbles dispersed in the slugs move with an average velocity u_{G3}. The void fraction within the slug at station 3 is ϵ. The slower moving liquid in front of the slug is accelerated to the average liquid velocity u_{L3} within the slug. The liquid shed at the back decelerates under the influence of wall shear and forms a stratified layer behind the slug with height h_{L1}. The stratified flow between slugs consists of a large gas bubble, moving with an average velocity u_{G1}, and a liquid layer moving with an average velocity u_{L1}.

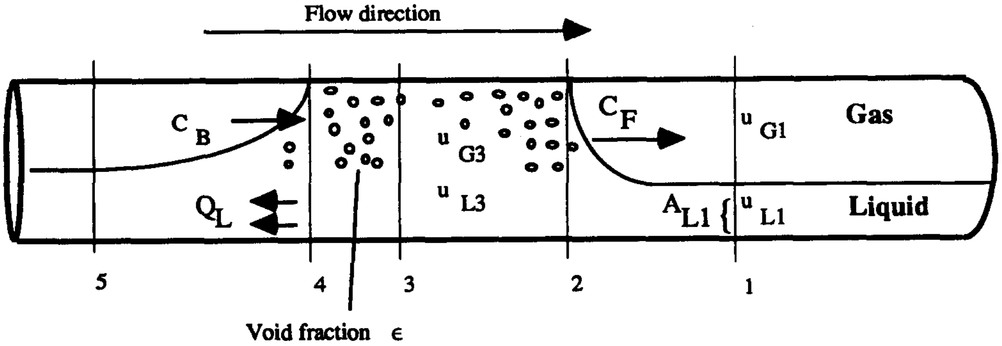


Figure 1. Definition of variables for a slug.

2.1. Volumetric flow rate of liquid out the tail of a slug

In a reference frame translating at the velocity of station 4, the volumetric flow rate of liquid shedding from the tail of the slug Q_L is given as

$$Q_L = (C_B - u_{L3})(1 - \epsilon)A, \tag{1}$$

where A is the area of the pipe. Ruder *et al.* (1989) chose to model Q_L with an inviscid solution, given by Benjamin (1968), that assumes negligible effects of aeration ($\epsilon = 0$):

$$Q_L = 0.542A\sqrt{gD}, \tag{2}$$

where D is the diameter of the pipe and g is the acceleration due to gravity.

Slug stability requires that the volumetric flow of liquid entering the slug at station 2, $(C_F - u_{L1})A_{L1}$, equals the rate at which liquid is shed at station 4, Q_L. By considering a steady state mass balance between station 2 and station 4 in figure 1, Ruder *et al.* defined a minimum liquid carpet height over which stable slugs can translate from the following equation:

$$\left(\frac{A_{L1}}{A}\right)_{\text{critical}} = \frac{Q_L}{(C_B - u_{L1})A}, \tag{3}$$

where $C_F = C_B$. Good agreement with eqn [3] was noted by Ruder *et al.* at low gas velocities, where aeration effects are small, if eqn [2] is substituted for Q_L.

By considering [3], it is clear that Q_L is a fundamental variable that is needed to understand slug flow. However, it is not established that a model which considers the tail of the slug to behave as a bubble, is useful to predict the shedding rates in horizontal flows under all conditions.

2.2. Relation between C_B and the input flow parameters

The specification of bubble velocity C_B has received considerable attention because of its fundamental importance in developing a model for slug flow. Common practice in vertical pipes is to relate C_B to be superficial liquid and gas velocities, U_{SG} and U_{SL}. Nicklin *et al.* (1962) postulated that the bubble velocity for vertical slug flow is the sum of the centerline velocity of the liquid in front of the symmetric bubble and the drift velocity of the bubble in a stagnant liquid.

$$C_B = K_0 u_{L3} + C_\infty, \tag{4}$$

where K_0 is a coefficient and C_∞ is the drift velocity of a bubble in a stagnant liquid. Nicklin *et al.* determined that for turbulent liquid flow K_0 is approximately 1.2, the ratio of the centerline liquid velocity to the mean liquid velocity u_{L3} in front of the bubble. Collins *et al.* (1978) considered the liquid flow around the nose of a bubble rising through flowing liquid in a vertical pipe. From their inviscid analysis, C_B is given by

$$C_B = u_C + \sqrt{gD}\,\phi\left(\frac{u_C}{\sqrt{gD}}\right), \tag{5}$$

where u_C is the centerline liquid velocity in front of the bubble and the function ϕ depends on the actual liquid velocity profile. Collins *et al.* showed that [4] is a reasonable approximation to [5].

Equation [4] has been generalized for slug flows for all inclinations. Assuming incompressible flow, a volume balance between the inlet of the pipe and station 3 within the body of the slug gives

$$U_{SG} + U_{SL} = \epsilon u_{G3} + (1 - \epsilon)u_{L3}. \tag{6}$$

If [6] is solved for u_{L3} and introduced into [4] the following equation for C_B is obtained:

$$C_B = C_0(U_{SG} + U_{SL}) + C_\infty, \tag{7}$$

where

$$C_0 = \frac{K_0}{1 + (s - 1)\epsilon}$$

and s is the slip between the liquid and gas phases within the slug ($s = u_{G3}/u_{L3}$). For horizontal flow $C_\infty = 0.542\sqrt{gD}$. Many authors neglect the drift velocity C_∞. Therefore, expressions such as $C_B = 1.35\,(U_{SL} + U_{SL})$ (Gregory & Scott 1969) and $C_B = 1.25$ to $1.28\,(U_{SG} + U_{SL})$ (Dukler & Hubbard 1975) are commonly used.

3. EXPERIMENTS

The experimental flow facility used in this investigation consists of a horizontal pipeline, with a diameter of 0.0953 m and a length of 26.5 m, that is operated at atmospheric conditions. The gas and liquid phases, air and water, are combined at the beginning of the pipeline in a tee section with the liquid phase in the run and the gas phase entering from the top of the tee.

Measurements of the variation of the liquid holdup were obtained with a liquid conductance technique used previously by Lin (1985), Andritsos (1986), Williams (1990) and Fan *et al.* (1992). A conductance probe consists of two parallel 24 a.w.g. chromel wires that traverse the diameter of the pipe vertically. When a signal is applied to one of the wires, the conductance between the two wires is dependent upon the height of liquid between the wires. Each conductance probe is calibrated individually to compensate for differences in probe construction. A complete description of the electronics involved in determining the output signal, including a circuit diagram, may be found in Williams (1990).

Conductance probes were used at $L/D = 200$, 220 and 250. For the measurements of the slug velocity and the flow of liquid out the tail of the slug, two conductance probes are needed. A third probe is added in order to observe the changes in a slug as it progresses along the pipeline. The third probe is also used to obtain better measurements of C_B and Q_L by averaging results from the first and second probes and from the second and third probes. The sampling frequency of each probe was varied from 0.5 to 2.0 kHz, depending on the gas velocity.

Pressure pulsations associated with the passage of a slug were measured with a piezoresistive pressure transducer located 0.127 m downstream of the first conductance probe. The transducer was mounted flush with the wall so that no disturbances were introduced into the flow.

The range of flow conditions studied is shown in the flow regime map in figure 2. It is noted that data were taken well above the transition from stratified flow to slug flow. The frequency of slugging close to the transition region is extremely small. The experimental conditions were chosen so that more than one slug appeared in the pipeline at a given time. Conductance data were also collected for plug flows, shown at low values of U_{SG} in figure 2. In this regime, the plugs of liquid

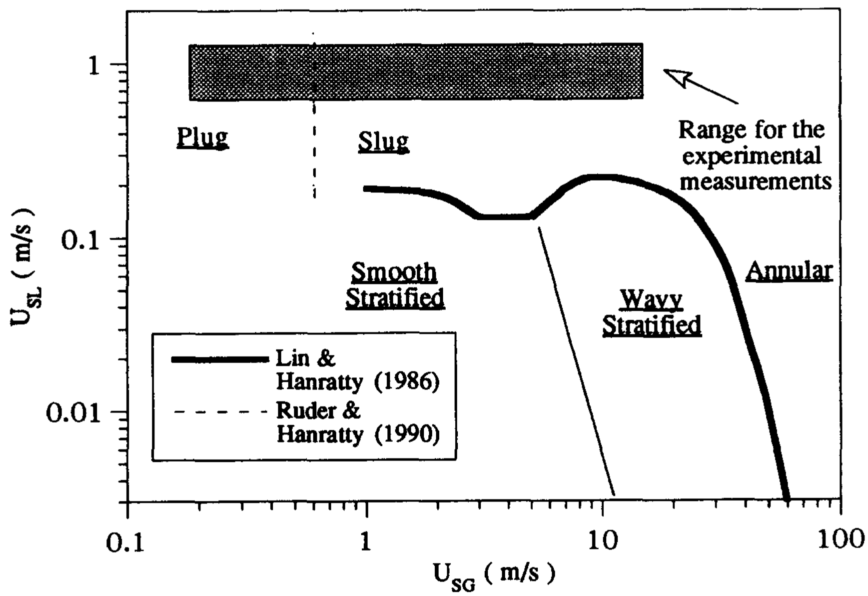


Figure 2. Flow regime map for air and water flowing in a horizontal 0.0953 m pipe at atmospheric conditions.

have negligible aeration, and the elongated bubbles between the plugs behave in a manner described by Benjamin's analysis (Ruder & Hanratty 1990). Ruder & Hanratty show that the transition from plug flow to slug flow is associated with the formation of a hydraulic jump. It is independent of U_{SL} and occurs at U_{SG} approximately equal to 0.6 m/s.

4. DETERMINATION OF LIQUID SHEDDING RATES OF SLUGS FROM CONDUCTANCE MEASUREMENTS

The conductance probes give the liquid holdup when a slug is present or the height of the liquid layer, h, when a stratified flow is present. Figure 3 plots these measurements as h/D versus time at $L/D = 200$, 220 and 250 for the flow condition of $U_{SG} = 6.5$ m/s and $U_{SL} = 0.7$ m/s. The wall pressure variations are also given. Spikes in the conductance profiles are caused by slugs that pass through the test section. The conductance profile of a slug does not reach the value of maximum conductivity ($h/D = 1$) due to the presence of bubbles. The seven slugs labeled in this figure are defined from a consideration of the pressure pulsations in figure 3d. The stratified flow between the slugs is observed to fluctuate in height; this contributes to the growth and decay of the slugs. For example, waves that are detected at $L/D = 200$ in front of slugs 6 and 7 are incorporated into these slugs, as indicated by the absence of the waves at $L/D = 250$. The aeration of slug 7 is observed to decrease after the incorporation of the wave.

Since the liquid carpet height between slugs is not constant, the volumetric flow of liquid out of the tail of individual slugs is expected to vary. Therefore, the shedding rates of an ensemble of slugs, calculated in this work, give the statistical characteristics of Q_L.

By constructing a control volume which translates downstream with the slug, values for the shedding rate, Q_L, of individual slugs are determined from conductance profiles at several locations along the pipe. Figure 4 shows an example of the construction of the control volume using the conductance profiles obtained with the upstream and downstream conductance probes. The rear of the control volume is attached to the rear of an individual slug, translating at velocity C_B. The front portion of the control volume is chosen to move with velocity C_B and is located at a position in the liquid carpet in front of the slug where changes in the liquid height are not observed between the two stations. Thus, the control volume is of constant length as it moves down the pipeline. The slug itself may be growing or decaying within the control volume. In reference to figure 1, the flow of liquid into the translating control volume at station 1 is $(C_B - u_{L1})\,A_{L1}$. The volumetric flow of liquid Q_L out the control volume at station 4 is $(C_B - u_{L3})\,A\,(1 - \epsilon)$. Conservation of volume for the liquid phase within the control volume requires that

$$Q_L = (C_B - u_{L1})\,A_{L1} - \frac{dV}{dt}, \qquad [8]$$

where dV/dt is the change in liquid volume inside the control volume with respect to time. The term dV/dt allows for the growth or decay of the slug.

The volumetric flow of liquid Q_L out of station 4 is determined by evaluating the right hand side of [8]. The slug velocity, C_B, is determined by dividing the distance between two probes by the time required for station 4 to travel between the probes. The height of the liquid at station 1, used to calculate A_{L1}, and the gas velocity u_{G1} (approximated by C_B) are used to calculate the liquid velocity at station 1, u_{L1}, with relations for a wavy stratified flow developed by Andritsos & Hanratty (1987). Errors in the estimation of u_{L1} are not serious for the flow conditions considered since $C_B \gg u_{L1}$. The derivative dV/dt is obtained by determining the volume of liquid within the control volume at the upstream and downstream probes. This is done at an instant in time by integrating the conductance profile from station 1 to station 4. The time derivative of the liquid volume within the control volume is given by:

$$\frac{dV}{dt} = \frac{C_B}{\Delta t_{12}}\left(\int_{\text{station 1}}^{\text{station 4}} \gamma(t)\, dt\,\Big|_{\text{downstream probe}} - \int_{\text{station 1}}^{\text{station 4}} \gamma(t)\, dt\,\Big|_{\text{upstream probe}}\right), \qquad [9]$$

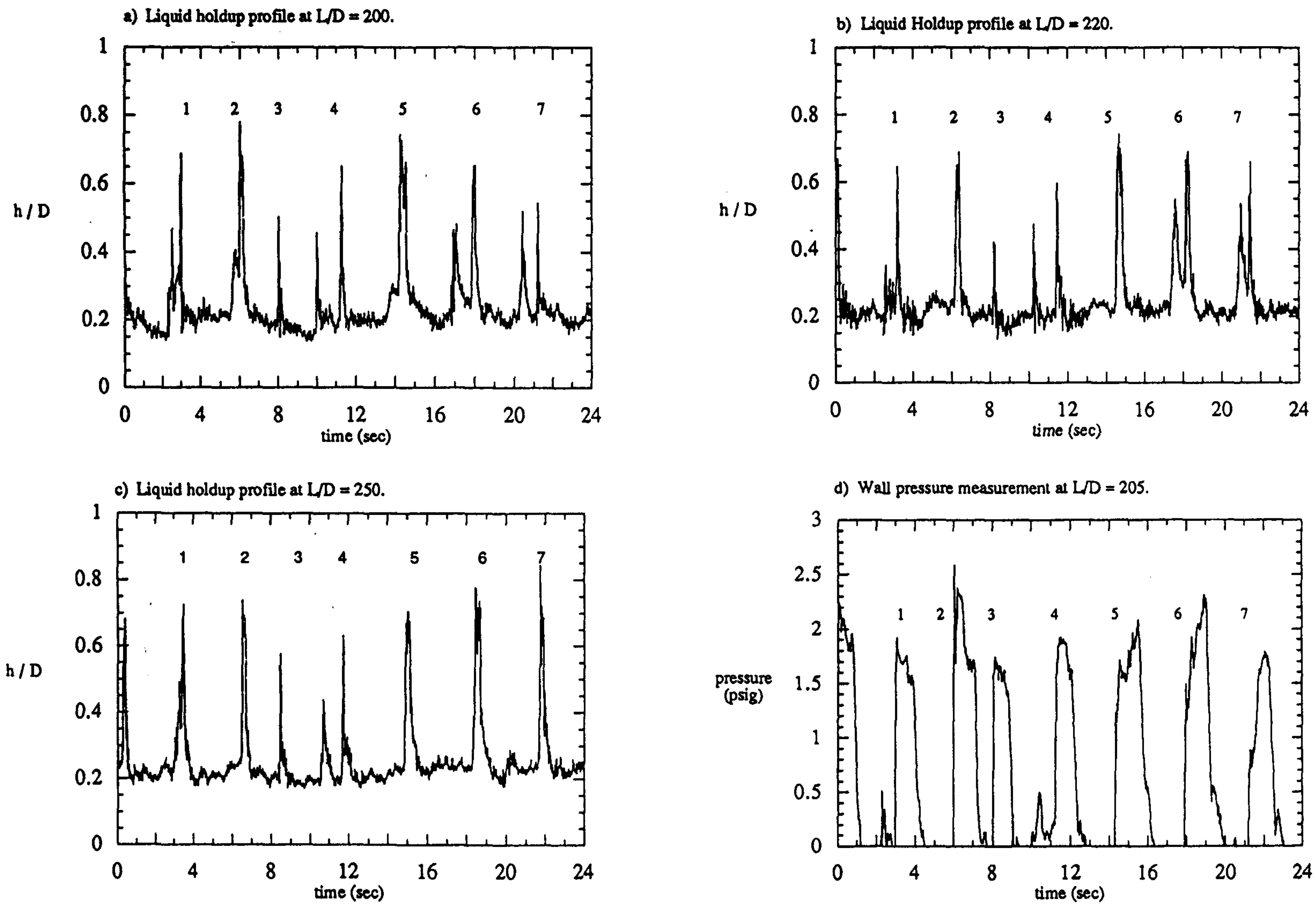


Figure 3. Conductance profiles and the pressure profile for slugs at $U_{SG} = 6.5$ m/s and $U_{SL} = 0.7$ m/s.

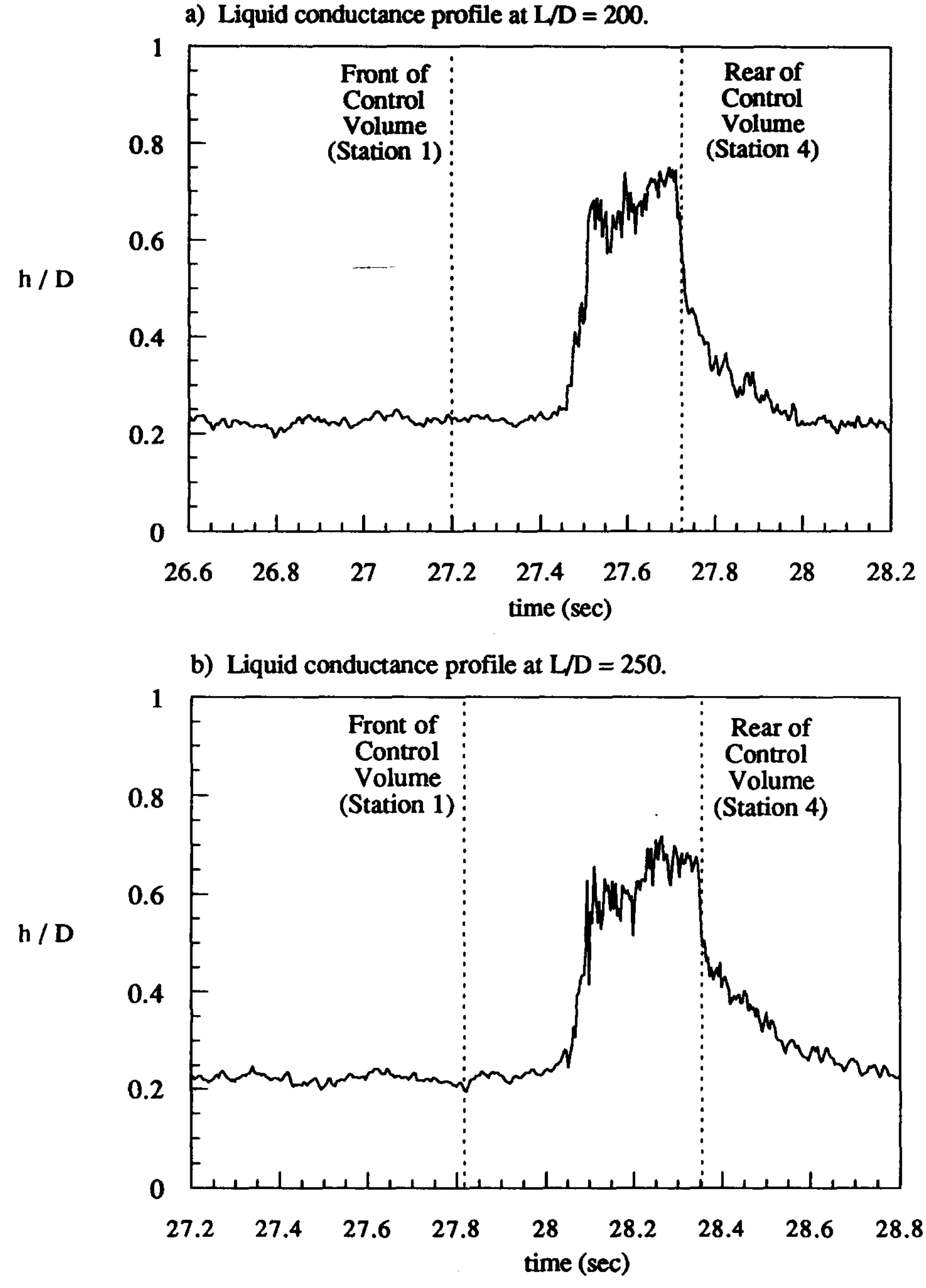


Figure 4. Example of the construction of a control volume attached to the tail of a slug (U_{SG} = 4.4 m/s, U_{SL} = 1.25 m/s).

where Δt_{12} is the time delay of station 4 between the upstream and downstream probes and $\gamma(t)$ is the liquid holdup profile within the control volume as it passes a probe. The flow of gas and liquid within the slug is considered to have a stratified configuration at low mixture velocities and a uniform configuration at high mixture velocities. The integrations were performed over individual slugs using Simpson's 3/8 rule.

The shedding rate is calculated twice for an individual slug, i.e. from profiles at $L/D = 200$ and 220, and from profiles at $L/D = 220$ and 250. These two values are averaged.

5. RESULTS

5.1. *Bubble velocity measurements*

For a given superficial gas velocity, the bubble velocity is observed to increase with U_{SG}, as shown in figure 5a. Local U_{SG} values are determined using the average pressure of the bubbles behind the slugs. In the limit of $U_{SG} = 0$ the bubble velocity is observed to approach the inviscid analysis of Benjamin (1968)

$$C_B = 0.542\sqrt{gD}. \qquad [10]$$

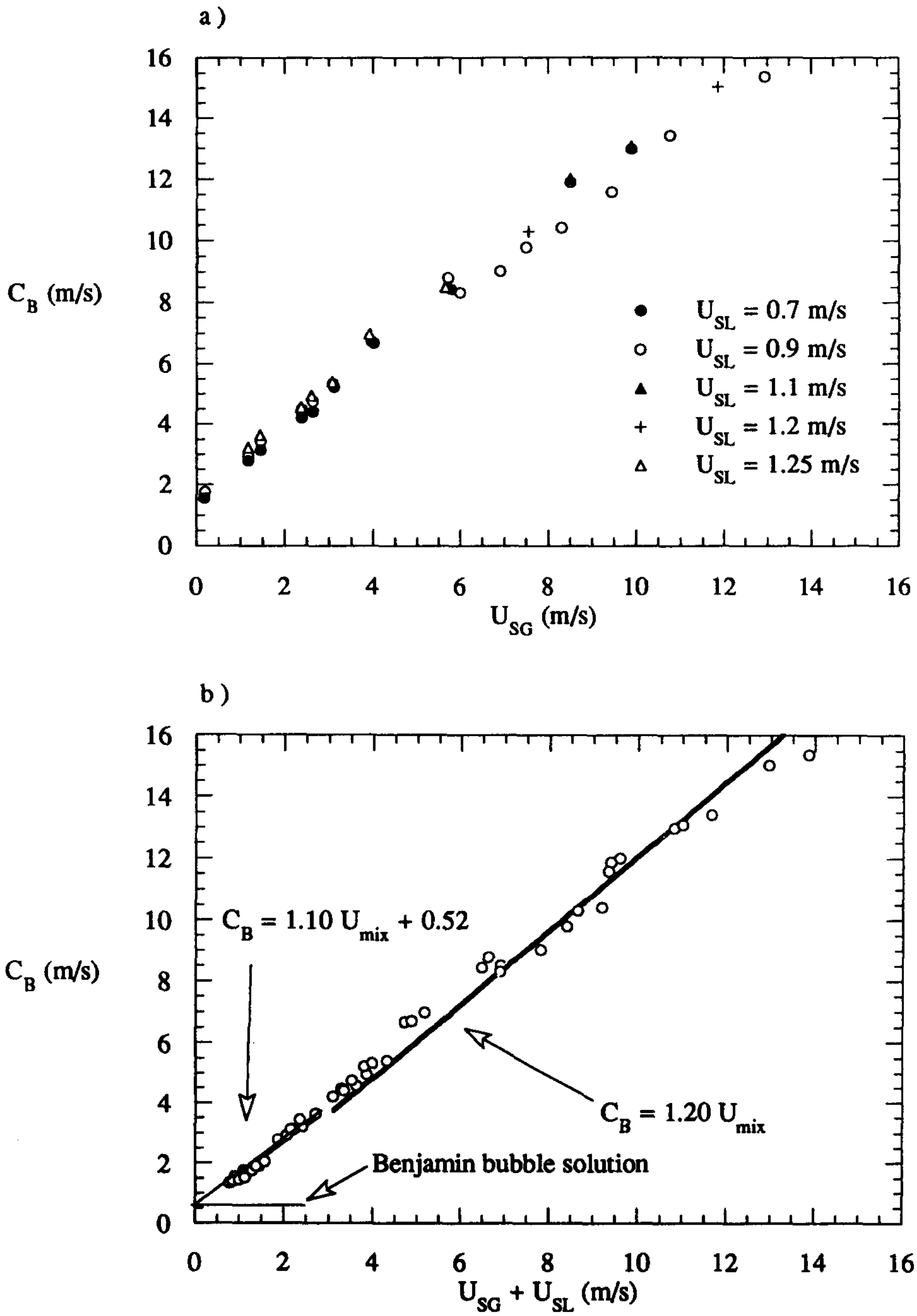


Figure 5. Measurements of the bubble velocity C_B as a function of (a) the local superficial gas velocity and (b) the local mixture velocity.

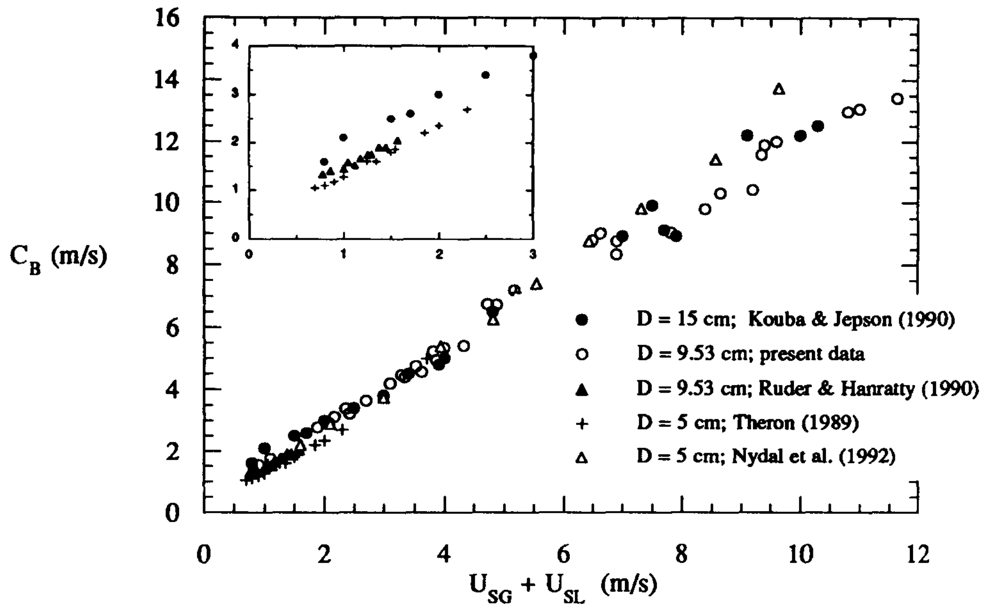


Figure 6. Effect of pipe diameter on the bubble velocity C_B.

The measurements of C_B in figure 5a are plotted in accordance with [7] in figure 5b. Values of C_B obtained by Ruder & Hanratty (1990) at low U_{mix} are included. These results suggest that the drift velocity contributes to C_B only for $U_{mix} < 3$ m/s, where $C_0 = 1.10$ and $C_\infty = 0.52$. This indicates that at low mixture velocities the bubble is not traveling with the centerline velocity of the liquid in front of it, for which case C_0 would equal 1.20. For $U_{mix} > 3$ m/s, the data are represented roughly with $C_0 = 1.20$ and $C_\infty = 0$. These results are in agreement with observations of the bubble centering process made by Bendiksen (1984).

Measurements of C_B obtained in this study with $D = 0.0953$ m are compared with measurements obtained by other investigators in figure 6. There is no evidence of an effect of pipe diameter at large mixture velocities. However, at small mixture velocities the bubble velocity increases with D.

5.2. Liquid holdup

The liquid holdup $(1 - \epsilon)$ was estimated for individual slugs from the liquid conductance profiles. At low gas velocities, visual observations suggested that the flow of gas and liquid in a slug resembles a stratified configuration; bubbles are buoyed to the top of the pipe. Thus, $(1 - \epsilon)$ is equal to the fractional area of the pipe occupied by the liquid. At high gas velocities, the bubbles within a slug are observed to be uniformly distributed over the entire pipe cross section. For this case, $(1 - \epsilon)$ is equal to the conductance ratio. Figure 7 compares the mean value of $(1 - \epsilon)$ at each flow condition with a correlation developed by Gregory *et al.* (1978) and a semi-empirical equation developed by Andreussi & Bendiksen (1989), which introduces a lower velocity limit, below which no bubbles exist in the plug or slug. Ruder & Hanratty (1990) have shown that the transition from plug flow to slug flow occurs at a gas velocity approximately equal to 0.6 m/s in a 0.0953 m pipe, independent of the liquid velocity. This gas velocity has been used as the lower velocity limit in the model of Andreussi & Bendiksen. Reasonable agreement between the experiment values and the prediction by Andreussi & Bendiksen is noted in figure 7. At high gas velocities, the mean liquid holdup values are expected to be lower than that of a fully developed slug flow. This is due to the existence of a large number of decaying slugs which are characterized by large void fractions.

5.3. Measurements of $\mathrm{d}V/\mathrm{d}t$

Figure 8 presents measurements of the mean value of the time rate of change of the amount of liquid within the control volume, $\mathrm{d}V/\mathrm{d}t$, for an ensemble of slugs for different U_{SG} and for a fixed $U_{SL} = 0.9$ m/s. The error bars in the figure designate two standard deviations about the mean of each ensemble. Included in this figure are measurements of the liquid volume found in an average

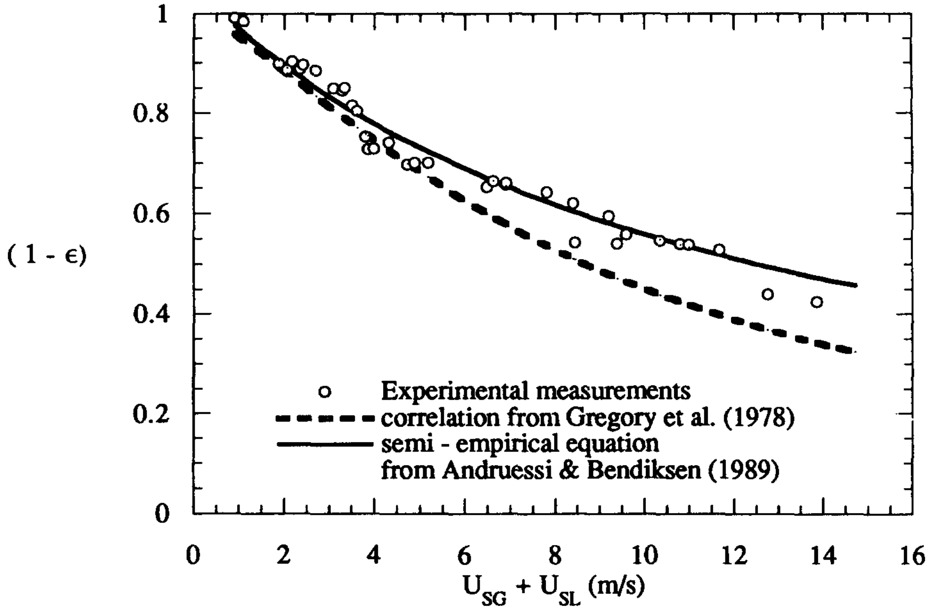


Figure 7. Liquid holdup measurements.

control volume. The mean value is approximately 0.012 m^3 at low gas velocities and decreases slightly at higher gas velocities. The mean value of dV/dt is approximately zero, but the standard deviation about the mean increases with gas velocity. For a given gas velocity, the mean values and the standard deviations of dV/dt for liquid flows of $U_{SL} = 0.7$ and 1.25 m/s show the same behavior as the data for $U_{SL} = 0.9$ m/s, shown in figure 8. The variations of dV/dt about the mean are approximately 0–10% of the liquid volume within the control volume for $U_{SG} < 2$ m/s, suggesting that the slugs at these gas velocities are relatively stable at $L/D = 250$. For $U_{SG} > 2$ m/s, the variations of dV/dt can be as high as 15–25% of the liquid volume inside the control volume indicating that many of the slugs are rapidly growing and decaying.

Figure 9 presents statistical distributions for the measurements of dV/dt at two different flow conditions. At the low gas velocity, $U_{SG} = 1.5$ m/s, the accumulation of liquid within a control

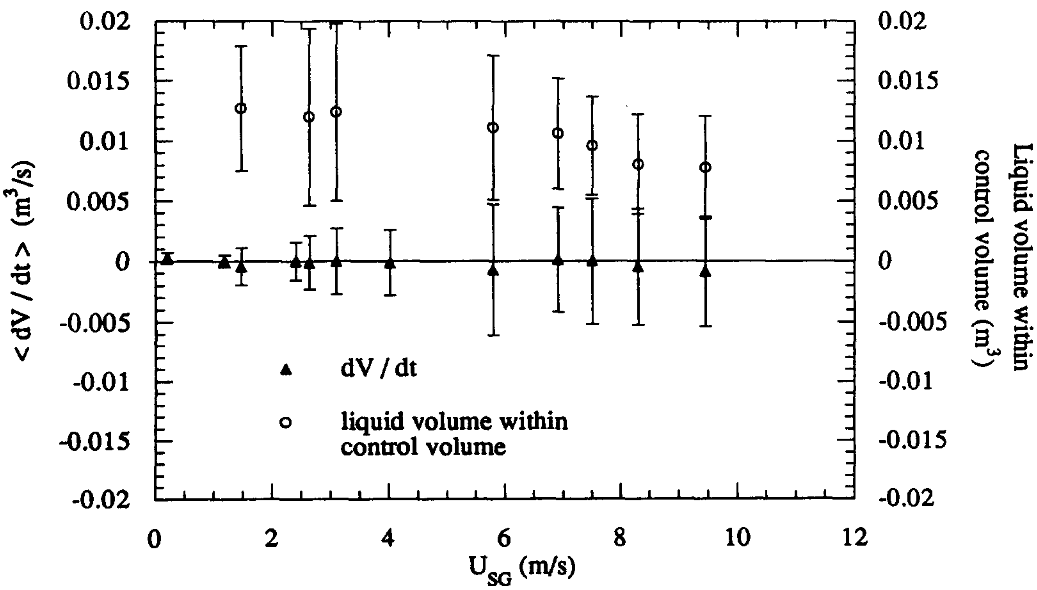


Figure 8. Measurements of dV/dt and the liquid volume within the control volume for $U_{SL} = 0.9$ m/s. Bars denote two standard deviations about the mean for each distribution.

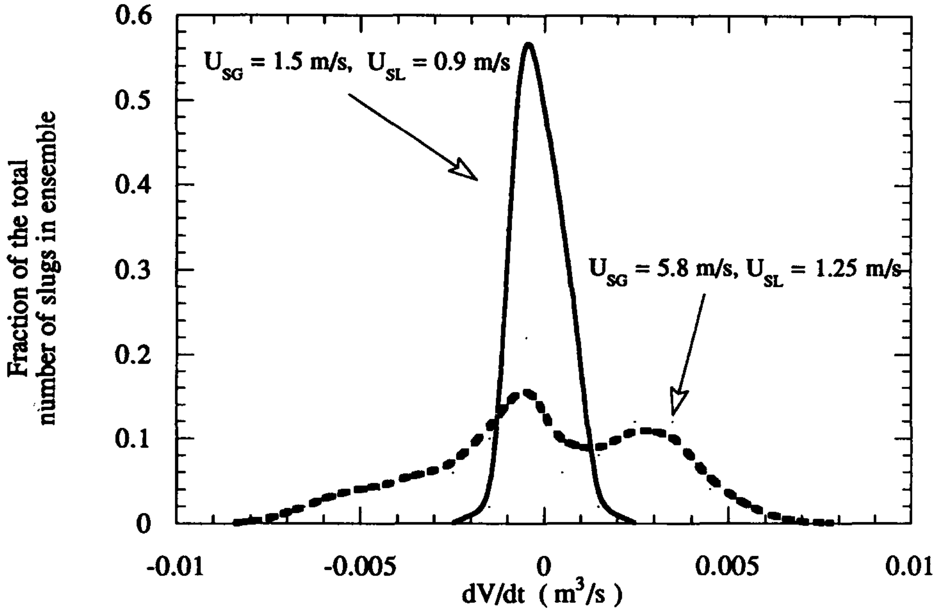


Figure 9. Distributions of dV/dt for two different flow conditions.

volume is narrowly distributed about $dV/dt = 0$. At this flow condition, a slug is initiated from the growth of a single wave by a mechanism described by Fan *et al.* (1993). The initiation of slugs is observed to occur approximately at the same location in the pipe for $U_{SG} = 1.5$ m/s. Thus, slugs at this gas velocity develop over an equivalent fetch and tend to possess the same characteristics once they reach the test stations. Because measurements are made far downstream from the point of initiation, the flow appears to be well developed at $L/D = 250$. At the higher gas velocity, $U_{SG} = 5.8$ m/s, growing and decaying slugs are distinguished by two separate peaks in figure 10. The large variability in the accumulation term at the higher gas velocity may reflect differences in the mechanism by which slugs are formed and the state of development at $L/D = 250$. Since slugs at high gas velocities ($U_{SG} > 4$ m/s) are formed by the coalescence of irregular, large amplitude waves (Kelvin–Helmholtz waves), they originate over a wide range of locations in the pipe (Lin & Hanratty 1986). Thus, the measuring station detects slugs at different stages in their development, since the fetch between the initiation point in the pipe and the measuring station varies from slug to slug. Furthermore, the bridging of the pipe due to the coalescence of waves is not always maintained; this results in large amplitude waves, that translate at lower velocities than the slugs. This failure of coalescing waves to grow into a slug is most likely due to an insufficient height of liquid in the base layer that supports a slug. Waves of this type are seen in front of the slugs labeled as 6 and 7 in figure 3. These eventually become incorporated into slugs.

Figure 10 presents an example of a decaying slug at three different locations in the pipe. The length of the slug in Figure 10a at $L/D = 200$ is 0.8 m. It decays to 0.5 m at $L/D = 250$ in figure 10c. The volume of liquid within the control volume varies from 0.0065 m^3 at $L/D = 200$ to 0.0052 m^3 at $L/D = 250$. This loss of liquid between the upstream and downstream probes corresponds to $dV/dt = -0.0023$ m^3/s. The volume of liquid in the tail appears to be greater at the two downstream probes than at the upstream probe, suggesting that the slug is shedding more liquid at the downstream probes. The liquid layer height in front of this slug is a constant value of $h_{L1}/D \sim 0.2$. Since the slug is decaying, this suggests that a liquid layer of this thickness cannot sustain a stable slug at this gas velocity.

5.4. Shedding velocities

At low gas flows, Ruder & Hanratty (1990) observed that the shedding velocity, Q_L/A, is consistent with the analysis of Benjamin (1968). As the mixture velocity increases, the calculation of the shedding velocity by the method outlined in section 4 shows that the tail of the slug deviates from the behavior of a Benjamin bubble in the manner shown in figure 11. As U_{mix} approaches

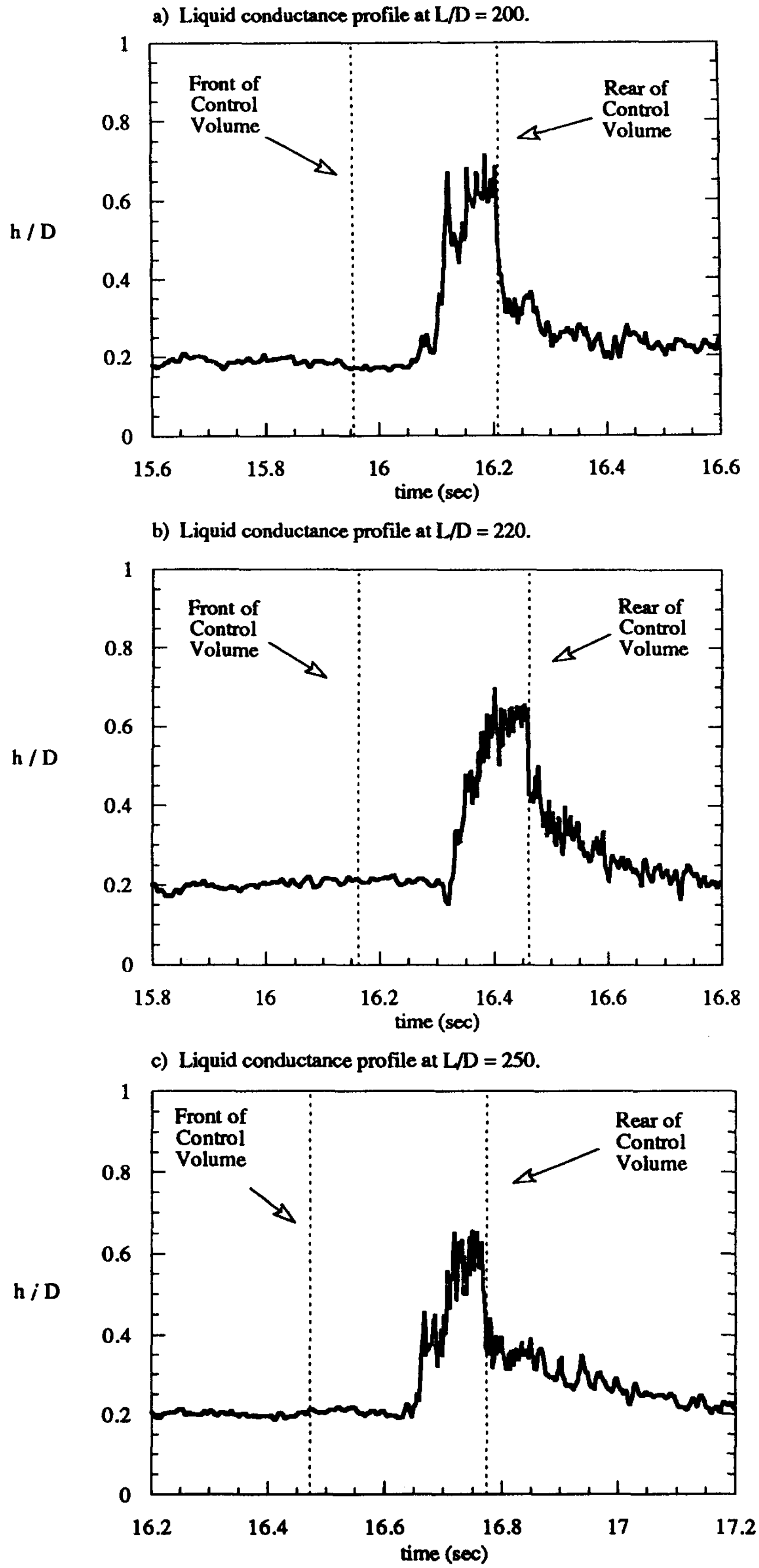


Figure 10. An example of a decaying slug at three different locations in the pipe ($U_{SG} = 5.8$ m/s, $U_{SL} = 0.9$ m/s).

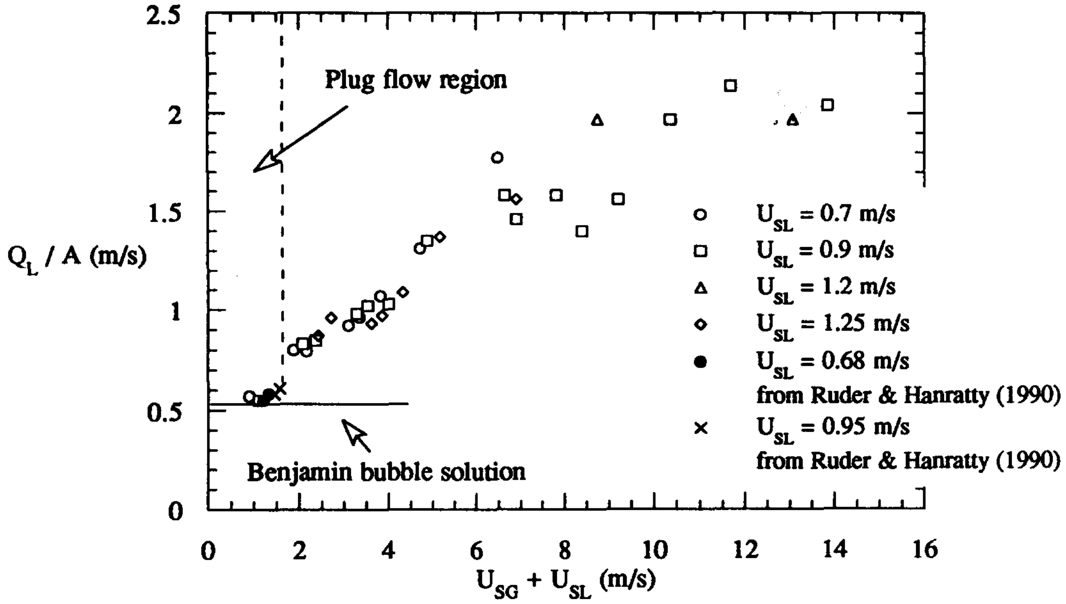


Figure 11. Measurements of the mean value of the shedding velocity Q_L/A.

0, it is seen that Q_L/A approaches $0.542\sqrt{gD}$, the inviscid solution of Benjamin. The value of the shedding velocity increases with U_{mix}. At the highest local mixture velocity, $U_{mix} = 14.2$ m/s, the volumetric flow of liquid out the rear of the slug is approximately four times that given by the Benjamin solution.

By solving [6] for u_{L3} and substituting this into [1], the following equation for the shedding velocity results:

$$\frac{Q_L}{A} = \left(C_B - \frac{U_{SG} + U_{SL}}{1 + (s-1)\epsilon}\right)(1 - \epsilon). \qquad [11]$$

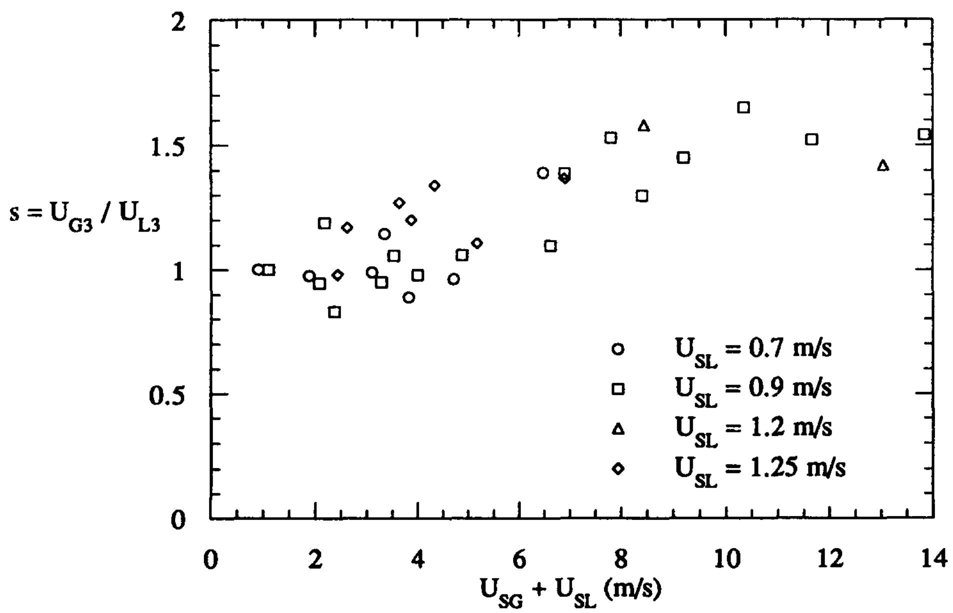
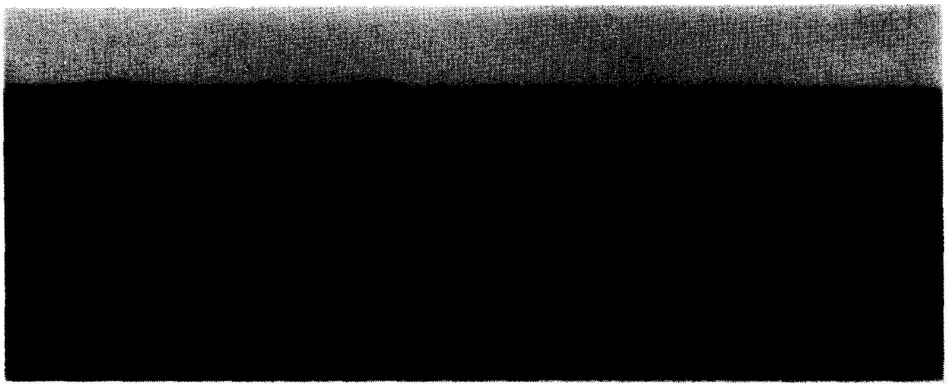


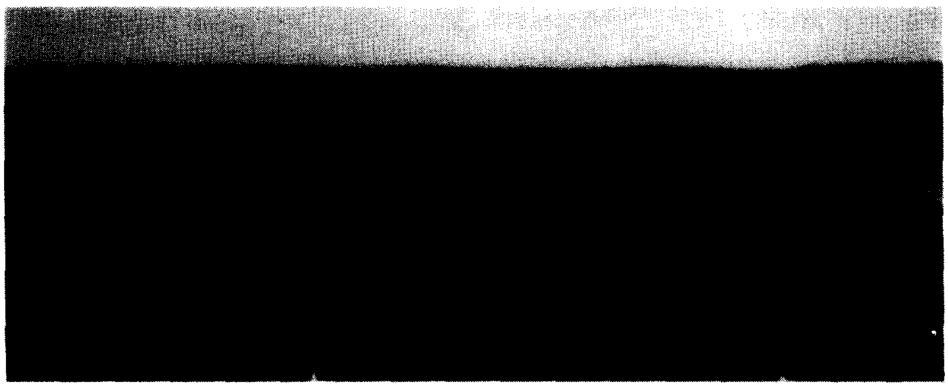
Figure 12. Values of the velocity ratio, s, needed to calculate the measured shedding velocity, Q_L/A, with [11].

Figure 12 gives velocity ratios ($s = u_{G3}/u_{L3}$) needed for the Q_L in figure 11 to be defined by [11]. Values of C_B from figure 5b, and values of $(1 - \epsilon)$ from figure 7 are used. A slip ratio equal to 1 is a rough approximation at low gas velocities. At high mixture velocities ($U_{mix} > 7$ m/s), a slip ratio of the order of 1.5 is indicated. It should be noted that the flow regime at high mixture

a) $U_{SG} = 0.2$ m/s, $U_{SL} = 0.9$ m/s



b) $U_{SG} = 1.5$ m/s, $U_{SL} = 0.9$ m/s



c) $U_{SG} = 2.4$ m/s, $U_{SL} = 0.9$ m/s

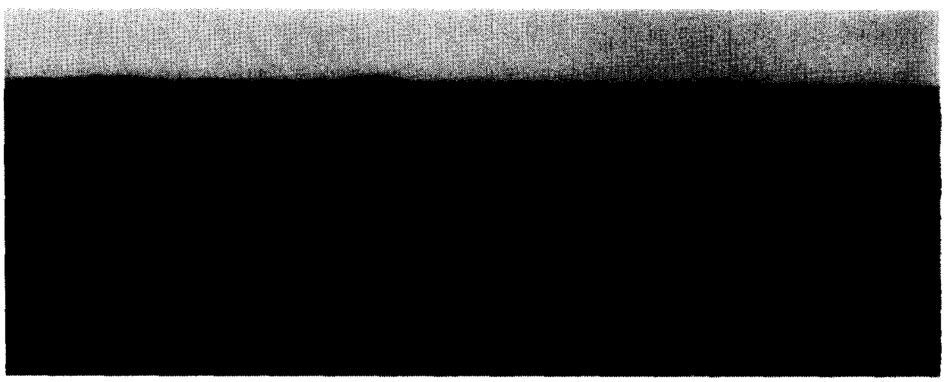
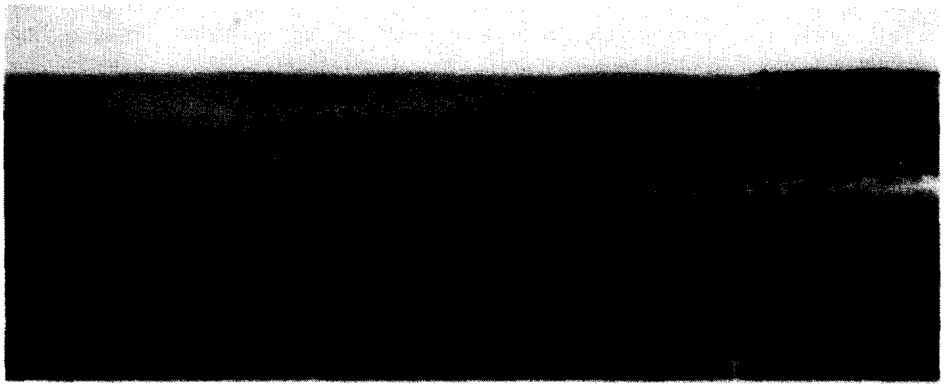


Fig. 13a, b and c

d) $U_{SG} = 3.0$ m/s, $U_{SL} = 0.9$ m/s



e) $U_{SG} = 5.5$ m/s, $U_{SL} = 0.9$ m/s

Figure 13. Photographs illustrating the typical behavior of the tail of a slug. The direction of flow is from right to left.

velocities is slug flow, and is distinguished from the pseudo slug flow regime by a characteristic pressure pulse. A slip ratio greater than one can be reconciled if the gas is blowing through the slug from the rear to the front. If the gas velocity is increased beyond 14 m/s, annular flow is observed.

The data for the shedding velocity in figure 11 (and the measurements of C_B in figure 5) suggest a change in the dynamics of the slug tail in the region of $U_{mix} = 3$ to 4 m/s. Since $\sqrt{gD}$ equals 0.97 for a 0.0953 m pipe, the values of Q_L/A in the region of $U_{mix} = 3$ to 4 m/s would correspond to a Froude number of one, indicating equal inertial and gravitational forces. A figure showing the relationship between a Froude number based on the shedding velocity and a Froude number based on the mixture velocity is not presented since data were obtained only for a single diameter pipe. At high gas velocities, the shedding velocity is dominated by inertial effects, so gravity plays a minor role. The results for the shedding velocity in figure 11 are consistent with a study of the behavior of single bubbles by Bendiksen (1984). Bendiksen suggested that a Froude number based on the mixture velocity

$$\left(\mathrm{Fr}_{mix} = \frac{U_{SG} + U_{SL}}{\sqrt{gD}}\right)$$

of 3.5 denotes a transition point for the dynamics of the bubble nose. For $\mathrm{Fr}_{mix} < 3.5$, the nose of the bubble was approximately located at the top wall, as assumed in Benjamin's theory (1968). Above $\mathrm{Fr}_{mix} = 3.5$, Bendiksen argues that the nose of the bubble becomes centered because inertial effects overcome gravity effects in the liquid in front of the bubble nose. In a reference frame moving with the bubble, the stagnation point on the liquid–gas interface therefore changes location from the top wall at small Fr_{mix} to a location somewhere between the wall and the centerline at large Fr_{mix}.

The photographs in figure 13 illustrate the behavior of the nose of the bubble as the gas velocity is increased from the plug flow regime through the slug flow regime. The flow direction in the photographs is from right to left. The gas bubble behaves as a Benjamin bubble for plug flow conditions, as noted by the location of a stagnation point approximately at the top of the pipe. The observed height of liquid beneath the bubble in figure 13a is in good agreement with the theoretical solution, which predicts $h_L/D = 0.56$ (Benjamin 1968). When U_{SG} is increased to 1.5 m/s, the entrained bubbles in the slug migrate to the top of the pipe due to buoyancy and form a thin layer, although the shape of the nose still resembles a Benjamin bubble. The values of Fr_{mix} for the slugs in figure 13c and d correspond to flow conditions where inertial and gravitational forces are of the same magnitude. A large fraction of liquid is shed out the top of the tail, suggesting that the stagnation point of the bubble nose is located closer to the center of the pipe. A large fraction of gas bubbles remain entrained in the shedding liquid resulting in the presence of bubbles in the liquid film immediately behind the slug; these bubbles eventually migrate out of the liquid so that the liquid layer between slugs does not contain much air. Figure 13e illustrates the uniform distribution of bubbles within the slug obtained at large gas velocities. Since inertial effects dominate, modeling the shedding velocity as a gravity current for this flow condition is clearly an approximation.

6. NECESSARY CONDITION FOR THE EXISTENCE OF A SLUG

6.1. Calculation of the stability height of a slug

Figure 11 shows that measured shedding rates agree with Benjamin's inviscid solution only for small U_{SG}. Consequently, the critical A_{L1} for the existence of a slug developed by Ruder *et al.* (1989),

$$\left(\frac{A_{L1}}{A}\right)_{\text{critical}} = \frac{0.542\sqrt{gD}}{(C_B - u_{L1})A} \qquad [12]$$

represents a lower limit. It is of interest to reexamine the analysis of Ruder *et al.* by using the present results. Equation [3] defines the critical A_{L1} for a stable slug. If [7] and [11] are introduced into [3], the following relation is obtained:

$$\left(\frac{A_{L1}}{A}\right)_{\text{critical}} = \frac{\left(\left(C_0 - \dfrac{1}{1+(s-1)\epsilon}\right)U_{mix} + C_\infty\right)(1-\epsilon)}{C_0 U_{mix} + C_\infty - u_{L1}}. \qquad [13]$$

For mixture velocities less than 3 m/s, the critical liquid layer height is determined using $s = 1$, $C_0 = 1.10$ and $C_\infty = 0.52$. For 3 m/s $< U_{mix} <$ 7 m/s, the stability height is determined using $s = 1$, $C_0 = 1.2$, $C_\infty = 0$. The velocity of the base film, u_{L1}, can be obtained from the stratified flow model of Andritsos & Hanratty (1987). For large mixture velocities, C_∞ and u_{L1} can be assumed to be negligible, so that [13] reduces to

$$\left(\frac{A_{L1}}{A}\right)_{\text{critical}} = \frac{C_0 - \dfrac{1}{1+(s-1)\epsilon}}{C_0}(1-\epsilon). \qquad [14]$$

Values of $s = 1.5$ and $C_0 = 1.2$ can be used in [14] for $U_{mix} >$ 7 m/s. Values of h_{L1}/D calculated from [13] for $U_{mix} < 7$ and from [14] for $U_{mix} >$ 7 m/s are represented in figure 14 by the open symbols. These values represent the height of the base layer required for the existence of a stable slug. Slugs are predicted to grow for h_{L1}/D larger than the critical and to decay for h_{L1}/D smaller than the critical.

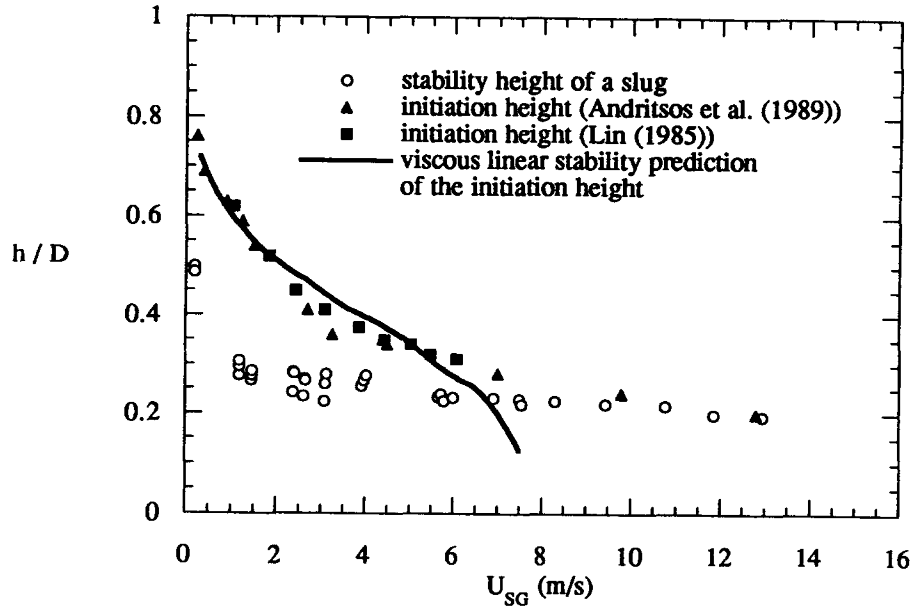


Figure 14. Comparison of the necessary conditions for a stable slug with the height of the stratified flow required to initiate a slug.

Examples of growing and decaying slugs are shown in figure 15 for $U_{SG} = 4.35$ m/s and $U_{SL} = 0.7$ m/s and for $U_{SG} = 9.45$ m/s and $U_{SL} = 0.9$ m/s. These are identified with positive and negative dV/dt in the control volume used to measure Q_L. A slug is defined to be stable if the volume of liquid within the control volume at a downstream location is within 5% of the volume of liquid at an upstream location. It is noted that decaying slugs are usually associated with values of h_{L1}/D lower than the critical.

6.2. Comparison of the necessary conditions with the height of liquid at transition

Figure 14 presents values of h_L/D measured by Andritsos *et al.* (1989) and by Lin (1985) at the transition from a stratified flow to a slug flow. The open symbols in figure 14 represent the necessary conditions for the existence of a slug calculated from [13]. These values represent a lower limit below which stable slugs cannot be generated. The theoretical prediction of the initiation height from a viscous linear stability analysis of a stratified flow (Lin & Hanratty 1986) agrees with the experimental data reasonably well at low gas velocities. At low U_{SG} the transition occurs at higher h_L/D than is needed for the existence of stable slugs. Under these conditions, slugs are observed to evolve from the instability of a stratified flow. At high U_{SG}, theories based on the stability of a stratified flow cannot explain the transition. It is noted in figure 14 that at high U_{SG} the h_L/D at transition agree with the h_{L1}/D needed for the existence of stable slugs. This suggests that transition at high U_{SG} is best interpreted by considering the stability of a slug rather than the stability of a stratified flow.

7. DISCUSSION

Measurements are presented for the shedding rate of slugs for the air–water system in a horizontal pipe. It is encouraging that these results are consistent with studies of stationary slugs by Fan *et al.* (1993) and with studies by Bendiksen (1984) of single bubbles injected into a water stream. Measurements and photographs support the notion that the back of a slug in horizontal flows can be modeled as a bubble for low mixture velocities. Air is incorporated into the slug at the front. For high mixture velocities, a slip ratio greater than unity is needed to interpret the measurements of the shedding rate. This suggests that air is moving through the slug from the rear to the front and that the tail of the slug may not be characterized as a bubble. Slip ratios are determined from [11]. This calculation is sensitive to the liquid holdup within the slug, $(1 - \epsilon)$. The measurements for the liquid holdup at higher gas velocities are expected to be lower than that of fully developed slug flow due to the existence of many decaying slugs at these gas velocities. Examples of these highly aerated decaying slugs are given in figure 15b. If larger values for the liquid holdup were used in [11] at high gas velocities, which may be more characteristic of fully developed slug flow, then slip ratios closer to 1 would be required in [11].

These measurements of shedding rates are used to predict the liquid carpet heights below which a slug will decay. Measurements of the carpet height in front of stable, growing and decaying slugs are consistent with this prediction. Of particular importance is the suggestion that the transition to slug flow at large gas velocities is best interpreted in terms of the stability of slugs rather than the stability of a stratified flow.

The curve describing the stability height in figure 14 is independent of the gas density. The prediction by stability theory of the critical height of the stratified flow needed to initiate a slug is a function of the gas density. Consequently, the theoretical curve representing the initiation height in figure 14 would be shifted to the left with increases in the gas density. Therefore, at very

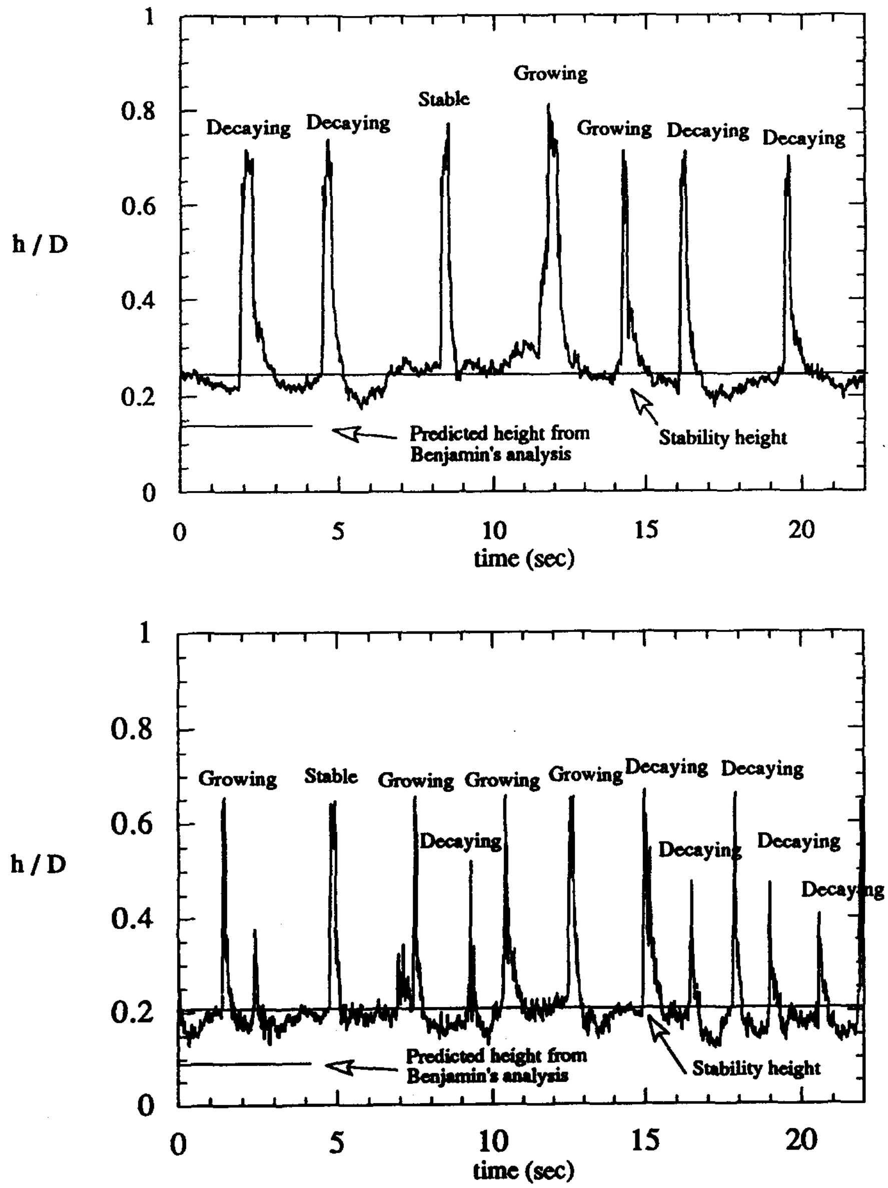


Figure 15. Examples of growing, decaying and stable slugs at two different flow conditions.

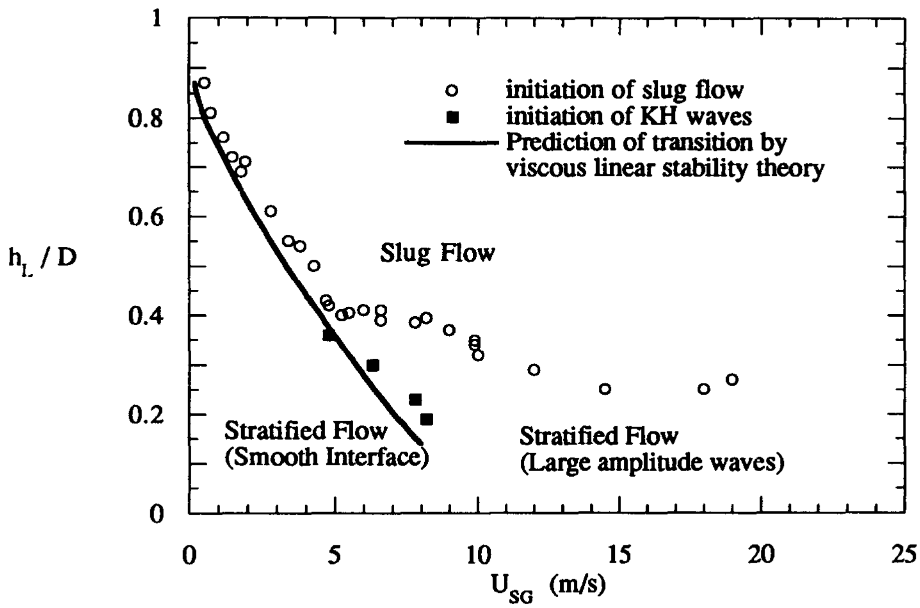


Figure 16. Initiation of slugs and Kelvin–Helmholtz waves for air flowing over a 100 cp liquid in a horizontal 0.0953 m pipe (Andritsos *et al.* 1989).

high gas densities the necessary conditions for the existence of a stable slug could be more restrictive than a stability analysis of a stratified flow.

It is of interest to compare figure 14 with measurements of the transition to slug flow for a 100 cp liquid in a 9.53 cm pipe (Andritsos *et al.* 1989). In this case slugs evolve from unstable capillary–gravity waves that are generated by a Kelvin–Helmholtz mechanism. For large h_L/D slugs are formed; for small h_L/D, the instability results in large amplitude waves. The solid curve in figure 16 is the Kelvin–Helmholtz instability. The circles represent the observed transition to slug flow. The filled squares represent a transition to large amplitude waves. The work presented in this paper suggests that the transition to slug flow, shown in figures 14 and 16, at large gas velocities should be interpreted by considering the stability of slugs, rather than the stability of a stratified flow. Andritsos *et al.* show that an increase in the liquid viscosity corresponds to an increase in the liquid layer height required for slug initiation. Thus, measurements of the stability height of slugs for a 100 cp liquid are needed to verify the suggestion that the transition to slug flow at large gas velocities is related to the stability of a slug.

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